Frequent Generalized Subgraph Mining via Graph Edit Distances

Richard Palme and Pascal Welke

IoT Streams for Predictive Maintenance | SeDaMi

UNIVERSITÄT BONN

A Motivating Use Case

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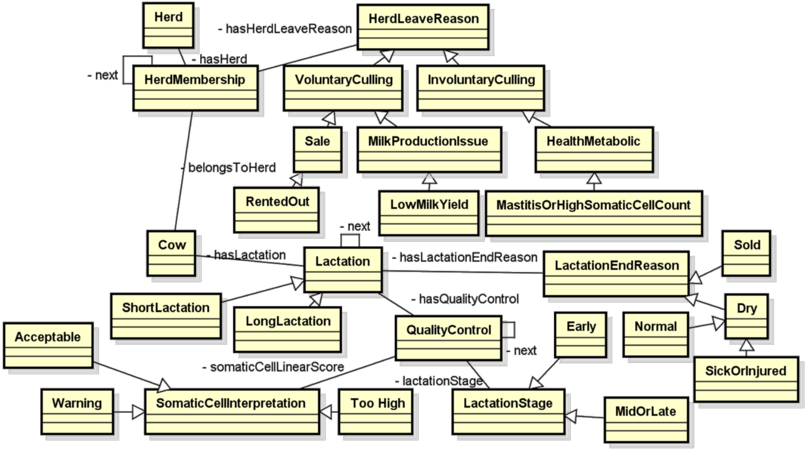
  - which traits to improve by selective breeding

A Dairy Cattle Performance Ontology



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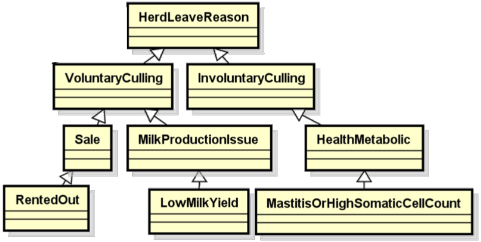

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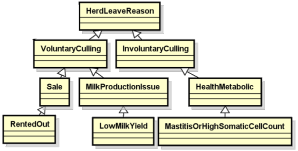

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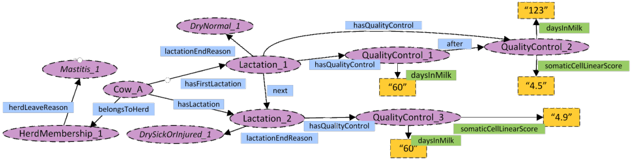

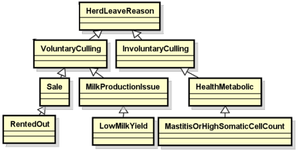

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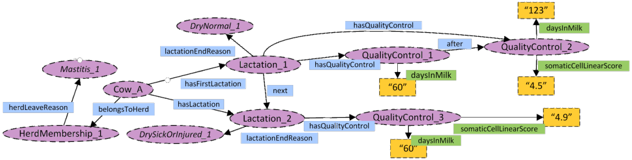

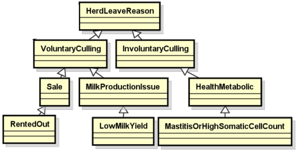

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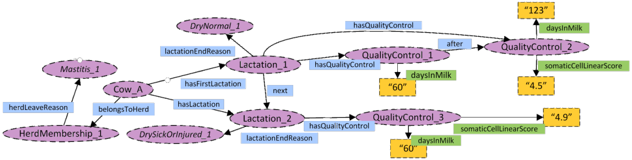

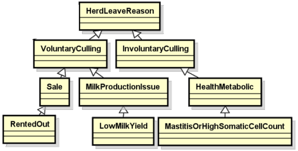

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- Are there some reoccurring patterns?

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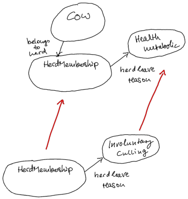

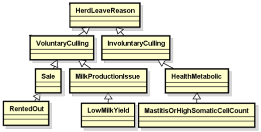
HerdLeaveReason
VoluntaryCulling
InvoluntaryCulling
Sale
MilkProductionIssue
HealthMetabolic
RentedOut
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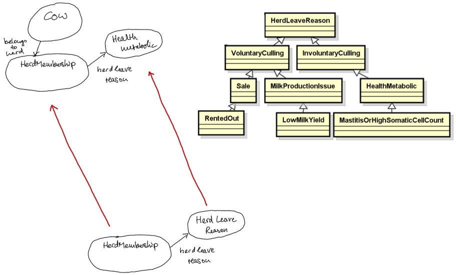

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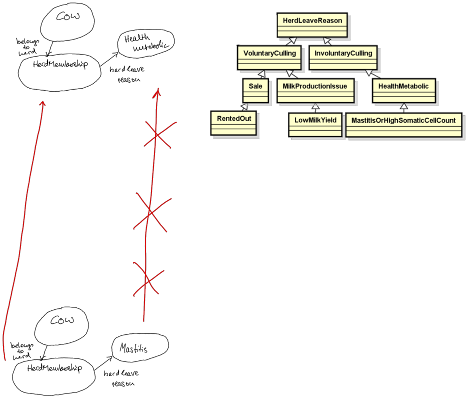

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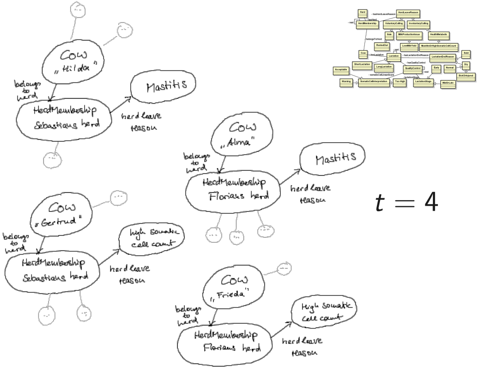


$t = 4$

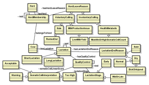
Generalized Subgraph Mining

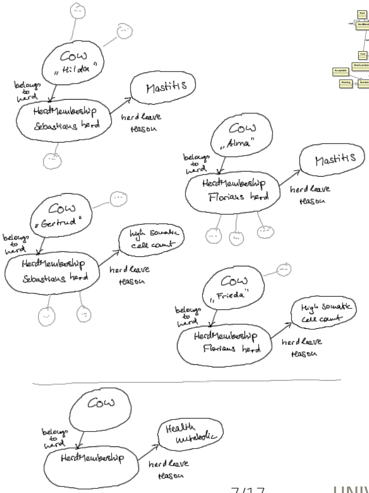
Frequent Generalized Subgraph Mining via Graph Edit Distances

The *Frequent Generalized Subgraph Mining Problem* is then:

Given: A database D of graphs, an ontology O and a frequency threshold t

Compute: The set of all graphs that are generalized subgraphs of at least t graphs in D





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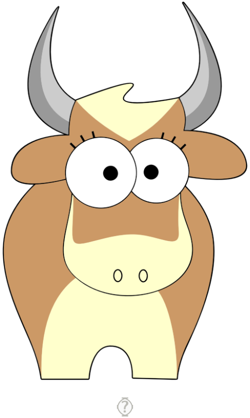
How Can This Help with Predictive Maintenance?

- Not automatically...



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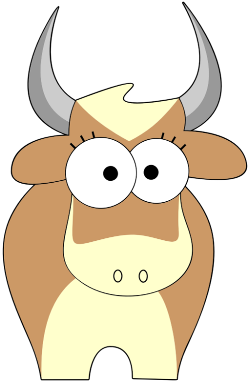

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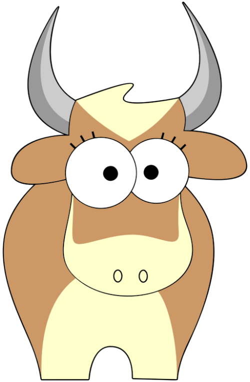
- But domain experts can interpret (smaller) frequent patterns
- And they can interpret changes in frequent patterns



Existing Approaches

Frequent Generalized Subgraph Mining has a long history

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These papers all modify classical frequent subgraph mining algorithms.

As a subroutine, they use subgraph isomorphism algorithms.

Our Approach

Proposal

We propose to replace subgraph isomorphism computations by *graph edit distance* computations

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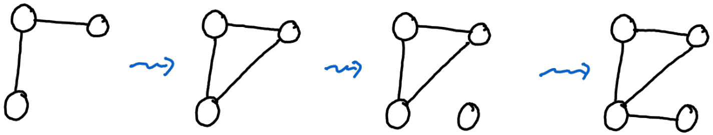
- But it allows some nice *freedom and simplicity* in modeling
- And it *simplifies* rather intricate mining algorithms

Graph Edit Distance

The *graph edit distance* between G and H is the smallest cost of a sequence of edits transforming G into H.



Graph Edit Distance

Edit operation	Edit cost
Insert an isolated vertex with label $\alpha \in \Sigma$	$c(\varepsilon, \alpha)$
Delete an isolated vertex u	$c(\lambda(u), \varepsilon)$
Substitute the label of a vertex u by $\alpha \in \Sigma$	$c(\lambda(u), \alpha)$
Insert an edge with label $\alpha \in \Sigma$	$c(\varepsilon, \alpha)$
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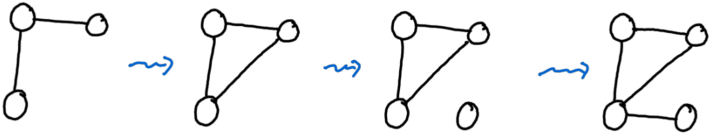

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The GED can be used to solve the *subgraph isomorphism problem (SGI)* by imposing the following three constraints on the edit cost function:

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$$\forall \beta \in \Sigma_\varepsilon \colon c(\varepsilon, \beta) = 0 \qquad \text{(free insertions)}$$

$$\forall \alpha \in \Sigma \colon c(\alpha, \varepsilon) > 0 \qquad \text{(paid deletions)}$$

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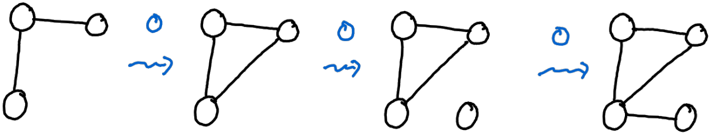

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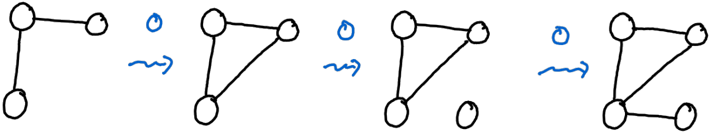

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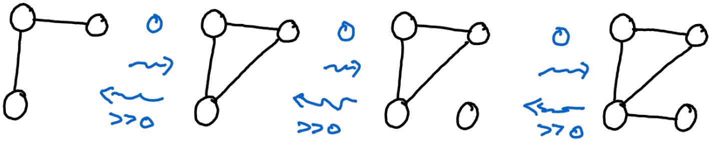

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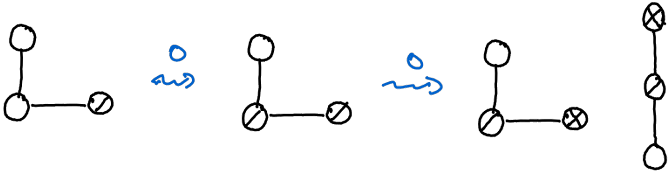

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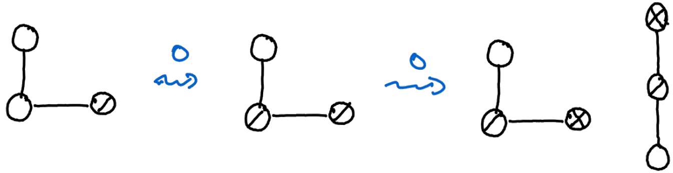

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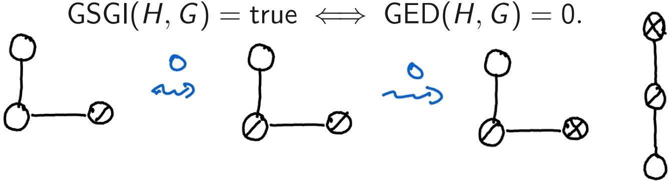

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- We say that a graph is a generalized subgraph if it has a small generalized subgraph edit distance

Implementation

- We implemented a proof of concept graph mining algorithm

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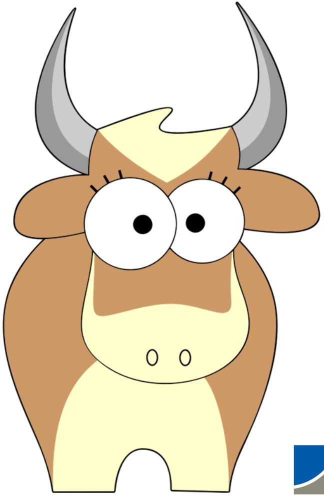
- It is available on `https://github.com/RichardPalme/fasm`

Conclusion

- Generalized Subgraphs can be identified with a specialized variant of the graph edit distance



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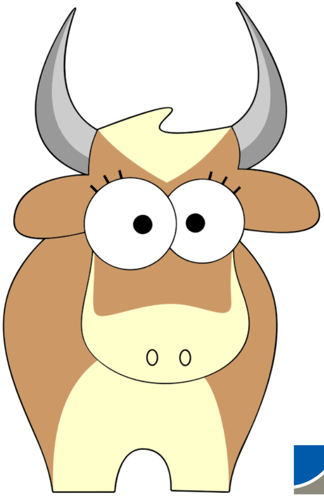

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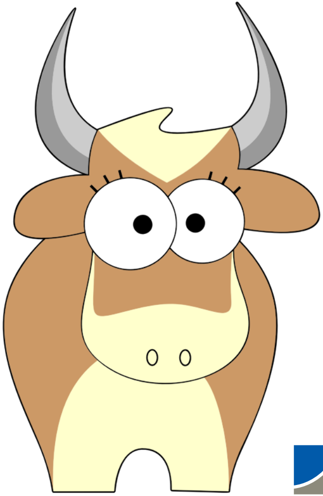
- We can include interesting costs (checkout the paper) to make the mining practically better



References I